Mental imagery in Computer Science

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The thesis

- Learning, understanding, memorizing,…

- Hence thinking

is facilitated by explicit construction of mental objects/representations and explicit manipulations through mental imagery.

Mental images versus real images

- Images, photos in the external world seem real but…they are not always (specially now):

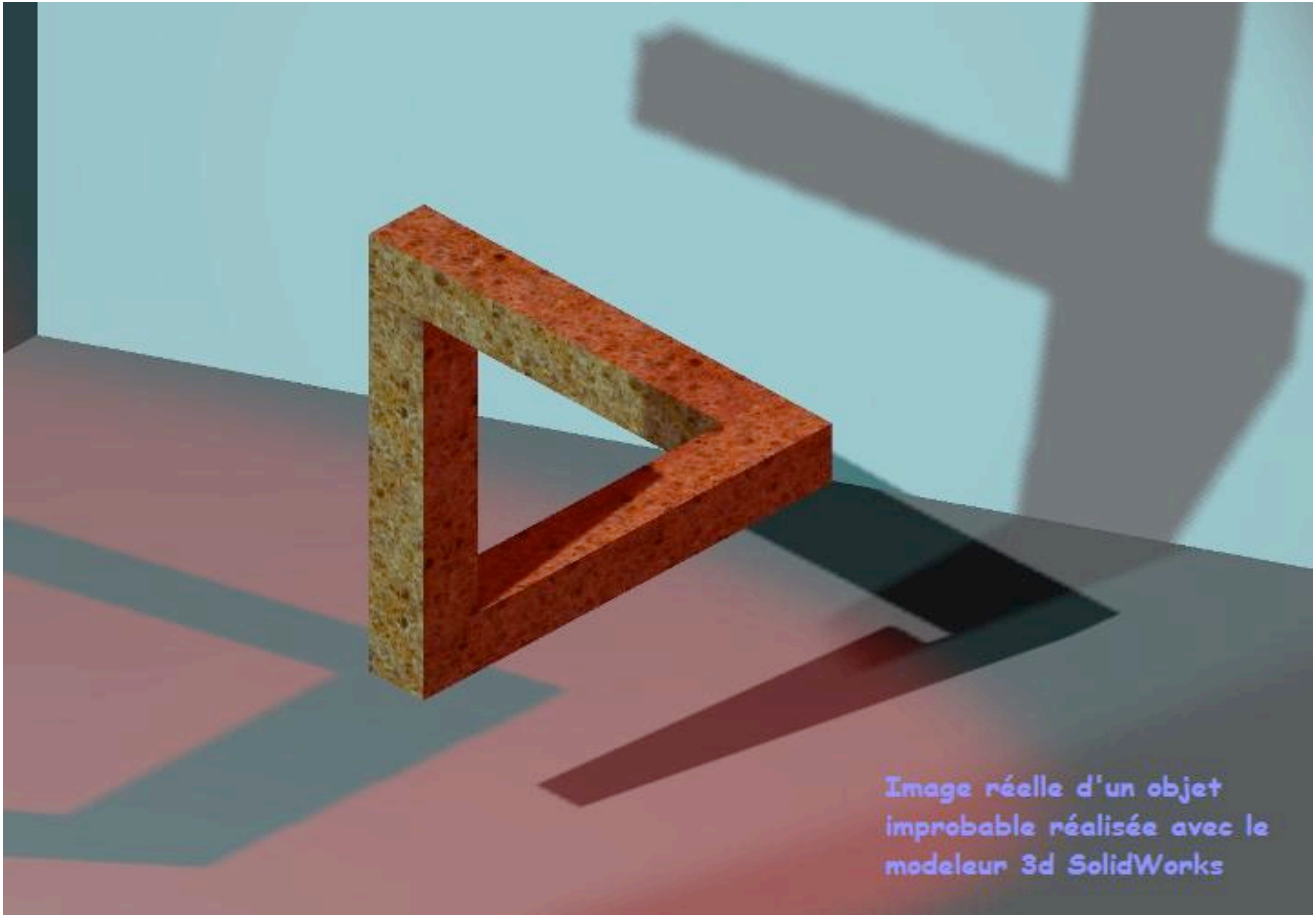


Image réelle d'un objet
improbable réalisée avec le
modeleur 3d SolidWorks

The reality of mental imagery…

Have we really images in the mind ? In the brain ?

May be, we have formula in the mind but we have the feeling to see something,…

Etc….

Philosophical questions with no definitive answer…

How to increase the quality of teaching, the knowledge of students and why not the quality of research communications ?

Attract more students for scientific studies…

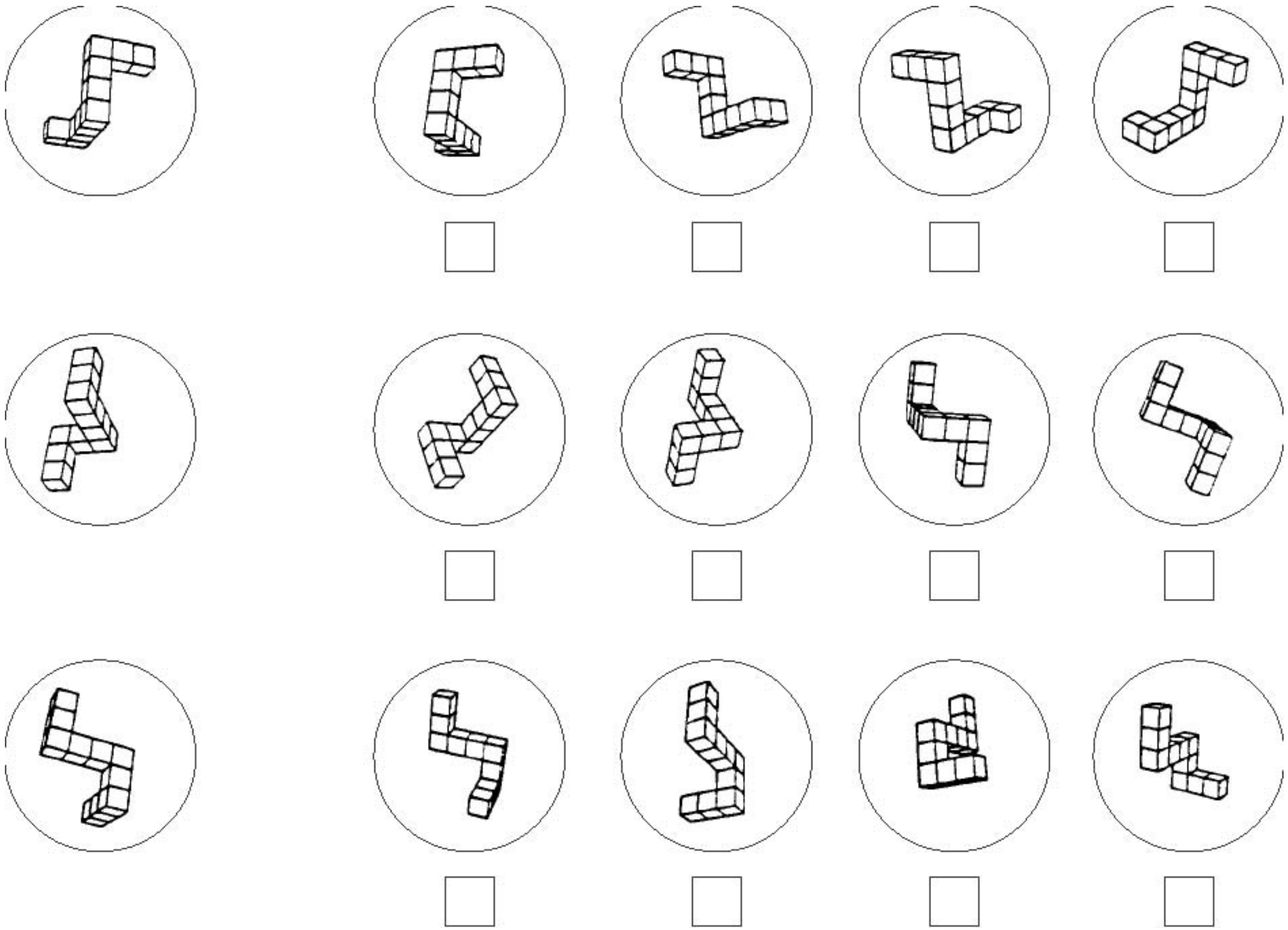
Plan

1. Mental imagery: generalities
2. Examples in mathematics
3. Examples in CS
4. The MI of genious research
5. Conclusions

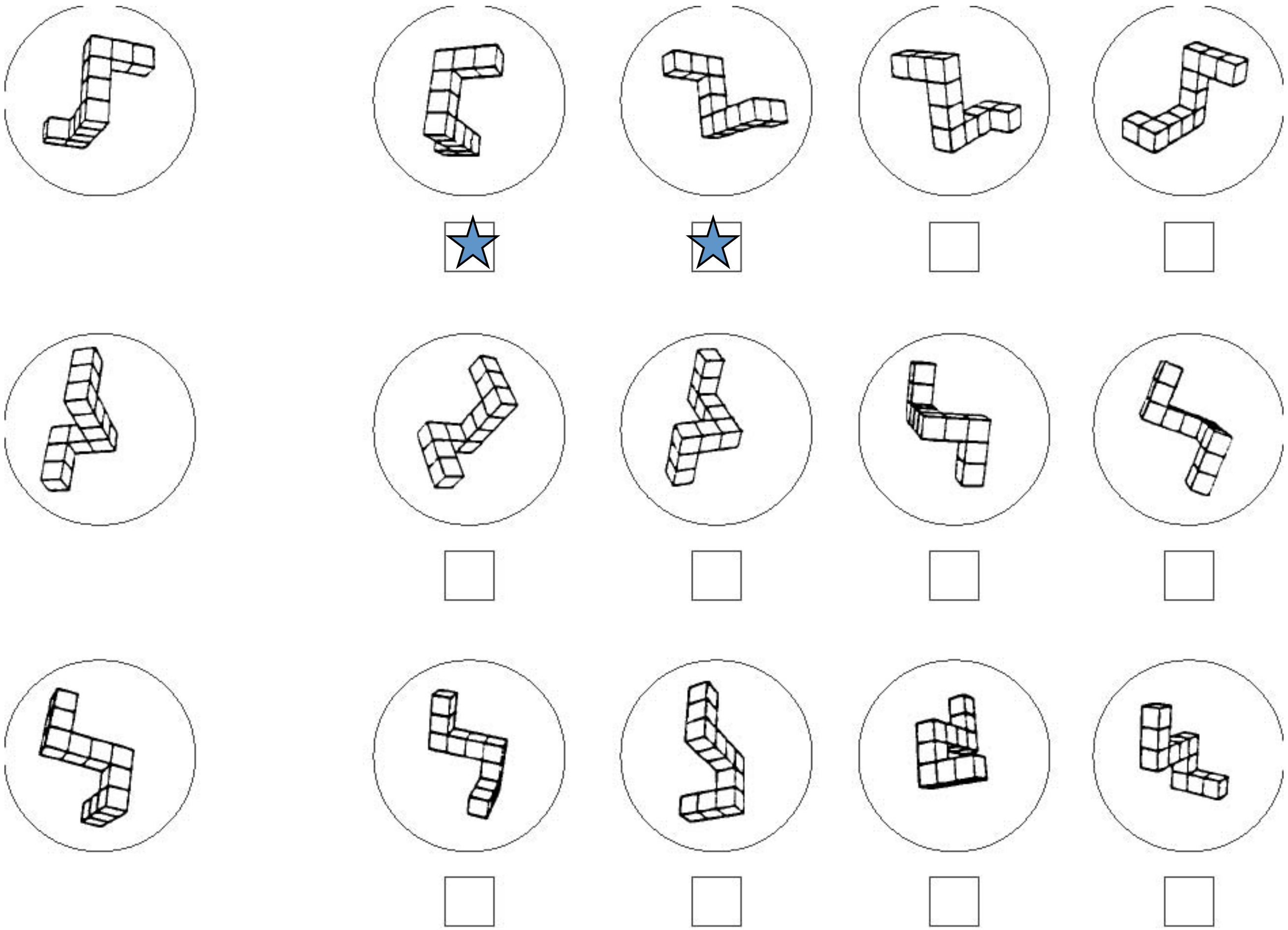
Fact 1: Mental Imagery "exist" and also some laws.

Partial proof: mental rotation (Vandenberg) and mental deplacements (Kosslyn)

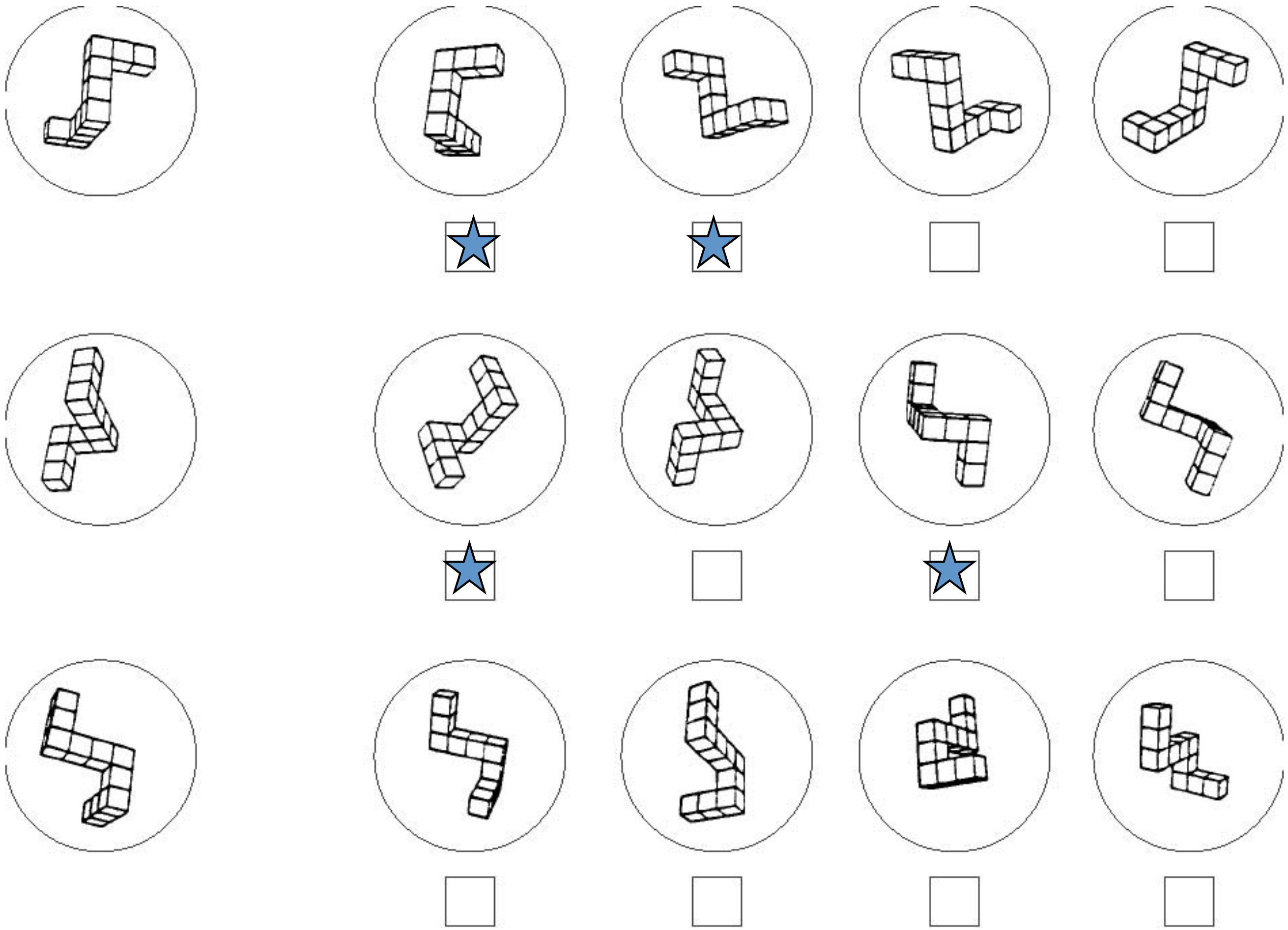
Test of mental rotation



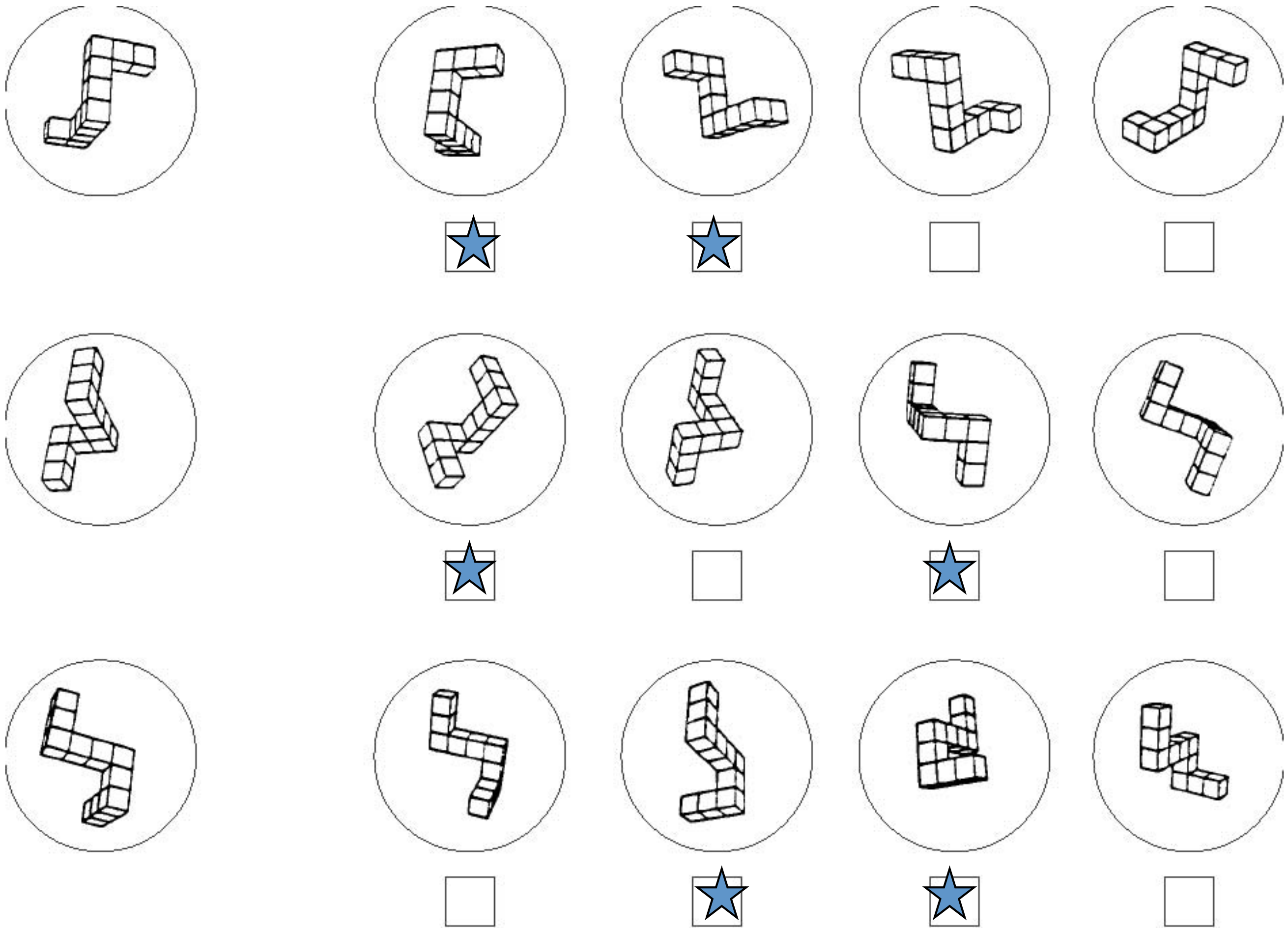
Test of mental rotation



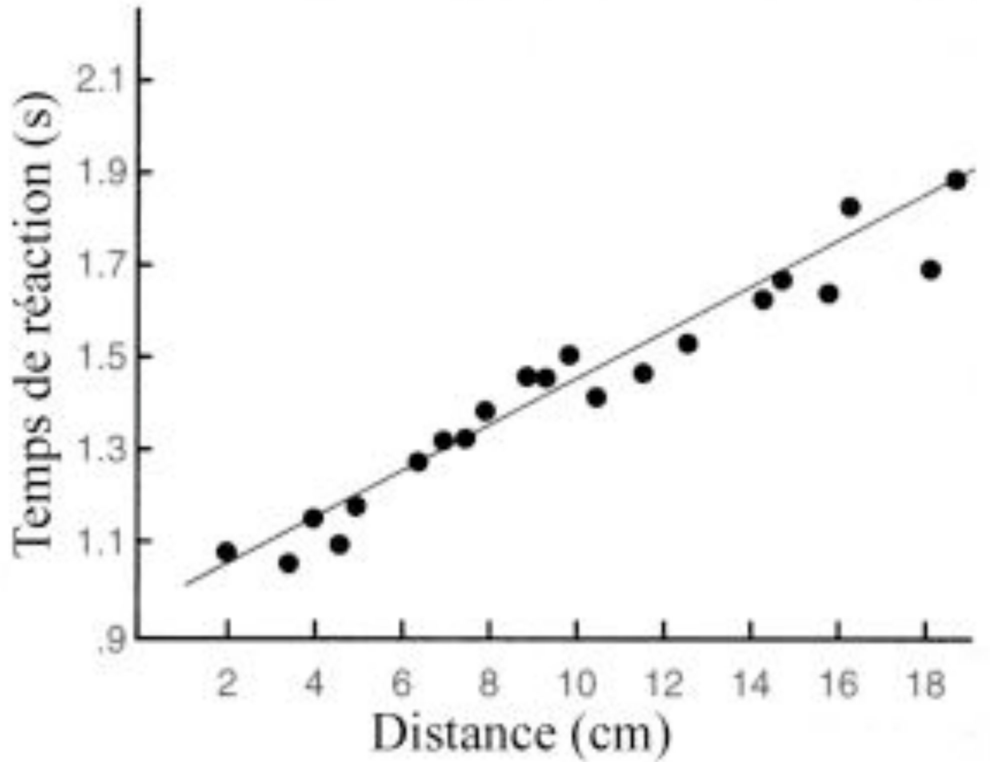
Test of mental rotation



Test of mental rotation



Kosslyn’s experience, 1978

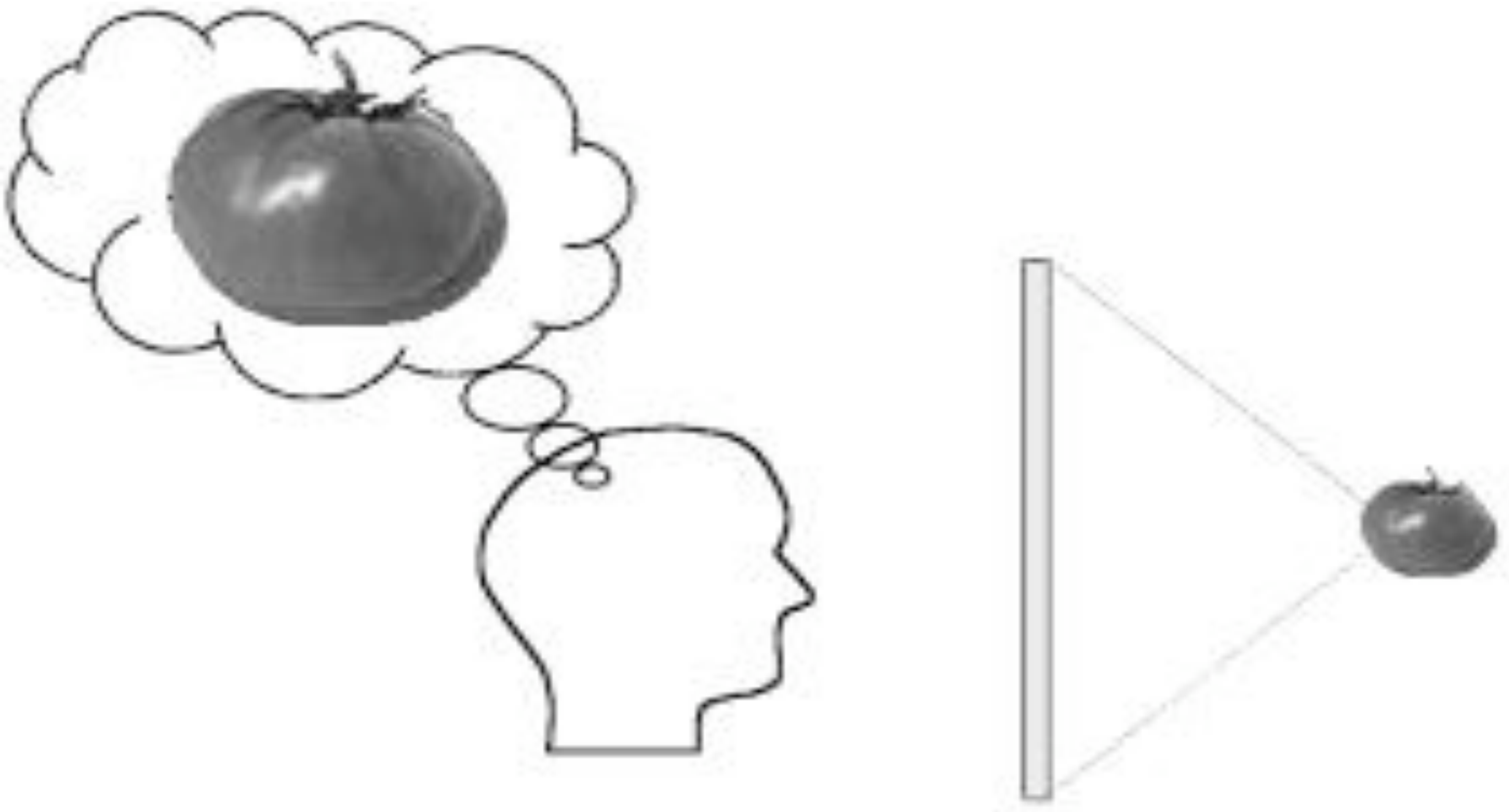


Fact: « mental speed » is constant

Fact 2: MI and perception share mental ressources

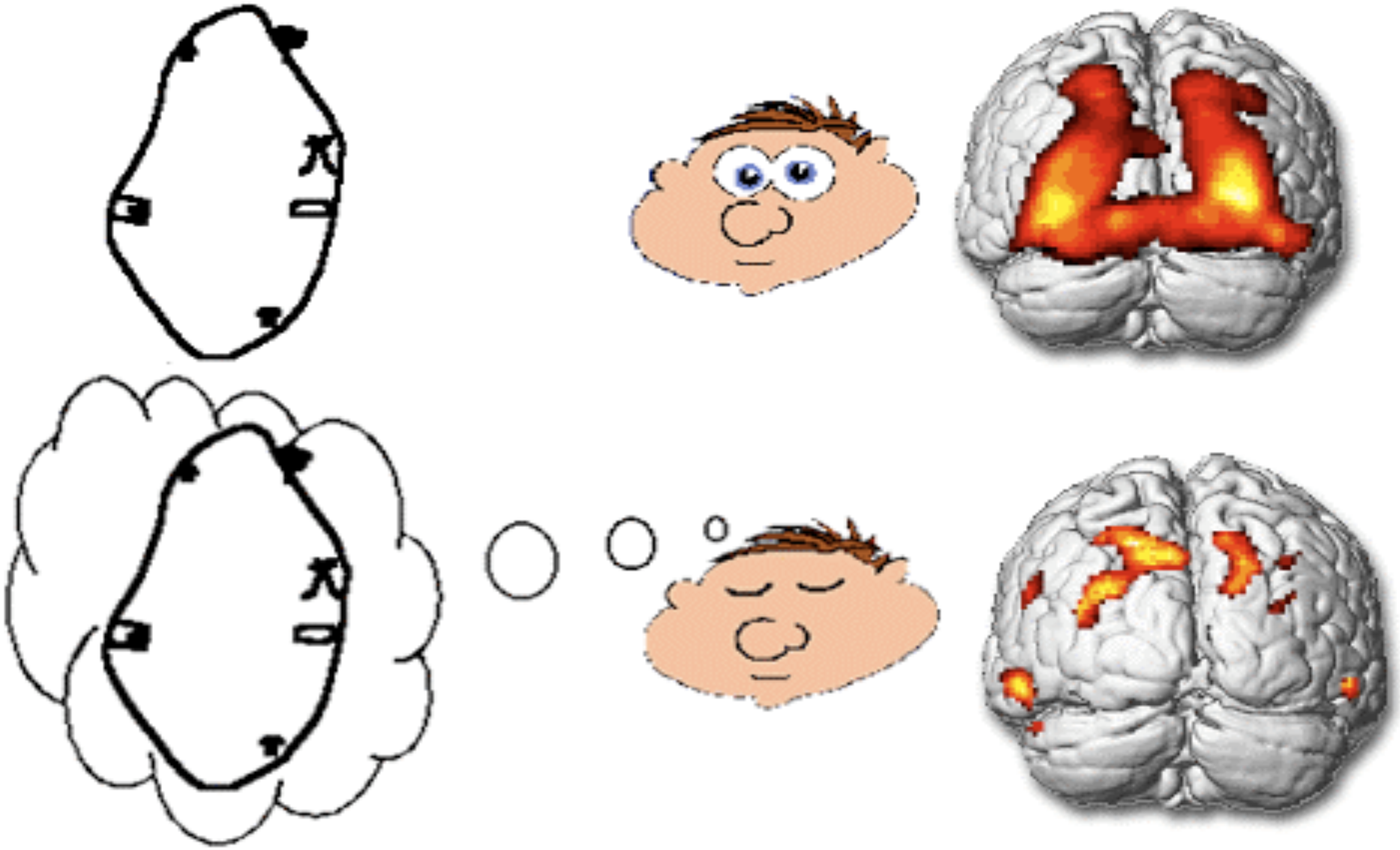
proofs: psychological & physiological

Perky's experience, 1910



Mutual exclusion: it is difficult to see outside and inside in the same time

Mellet’s experience, 1995



Observing the brain during MI and Perception: = and ≠

Dog, fly and elephant in the mind

- Imagine a dog, and then add a fly

  Time = c

- Imagine a dog and then add an elephant

  Time = c + 200 ms

- Kosslyn (1975)

proof: Paivio 1970

image+verbal > verbal

Pre-conclusion on MI

1. MI “exist”: mental rotation, mental speed.

2. MI versus perception. Mutual exclusion, mental resources, attention.

3. Double coding is better than unique one.

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Algebraic identities

- Theorem: $(a + b)^2 = a^2 + 2ab + b^2$

- Verbal proof:

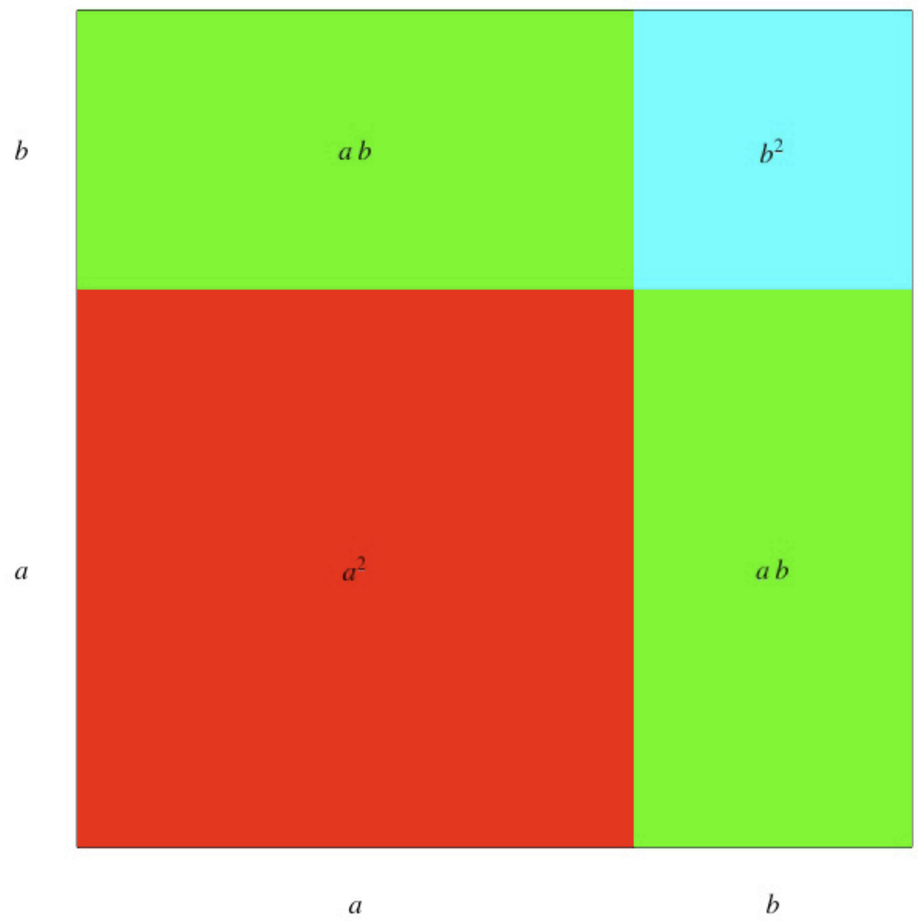
$(a + b)^2 = (a + b)(a + b)$ by definition

$= a^2 + ab + ba + b^2$ by the rules of computation

$= a^2 + 2ab + b^2$ by commutativity of multiplication

- Memorization: by memorizing the proof, by repetition of the melody, by seing it in the mind and outside,…

Visual proof



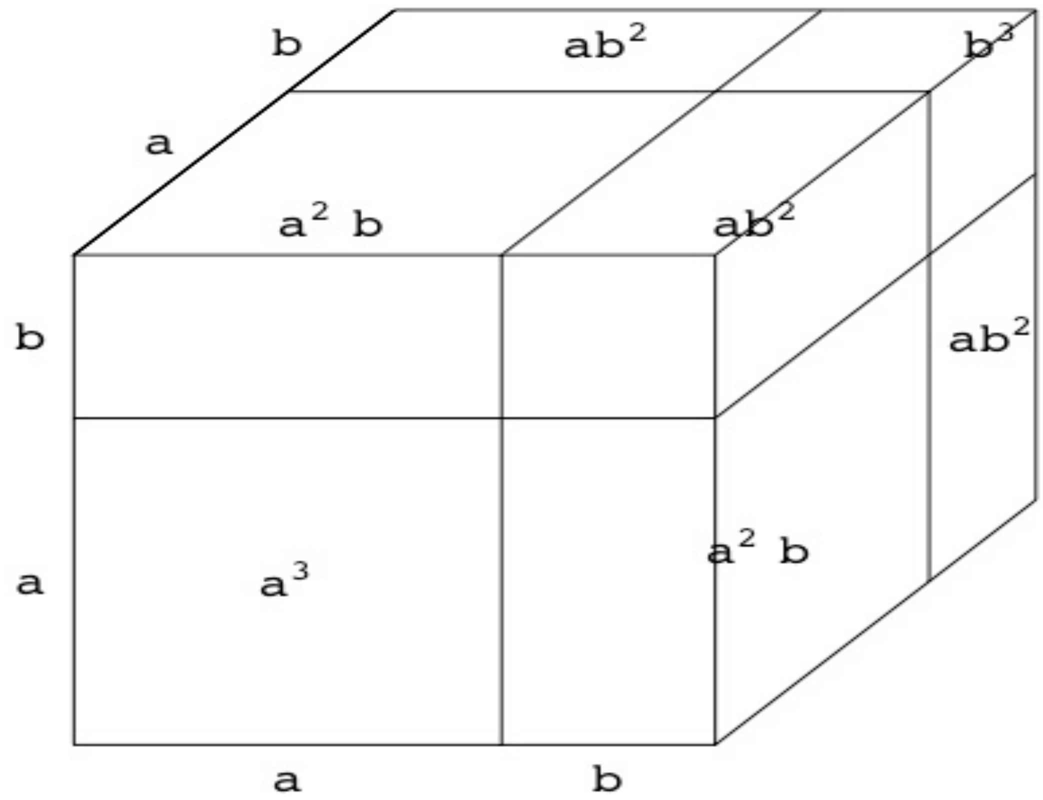
Concrete proposal

- Present to students the two mental objects (words and images) for increasing understanding and memorization.

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

- Verbal proof and verbal memorization: as usual, symbolic computation.
- Not so easy to memorize.

It is also possible to see and only after to compute



Verbal computation or visual manipulation ?

- At the beginning: words or images ?
- Difficult to know
- Let us use both.

$S(n) = 1 + 2 + 3 + \ldots + n$

- Proof of $S(n) = n(n+1)/2$ is possible by recurrence but with no intuition, no understanding.

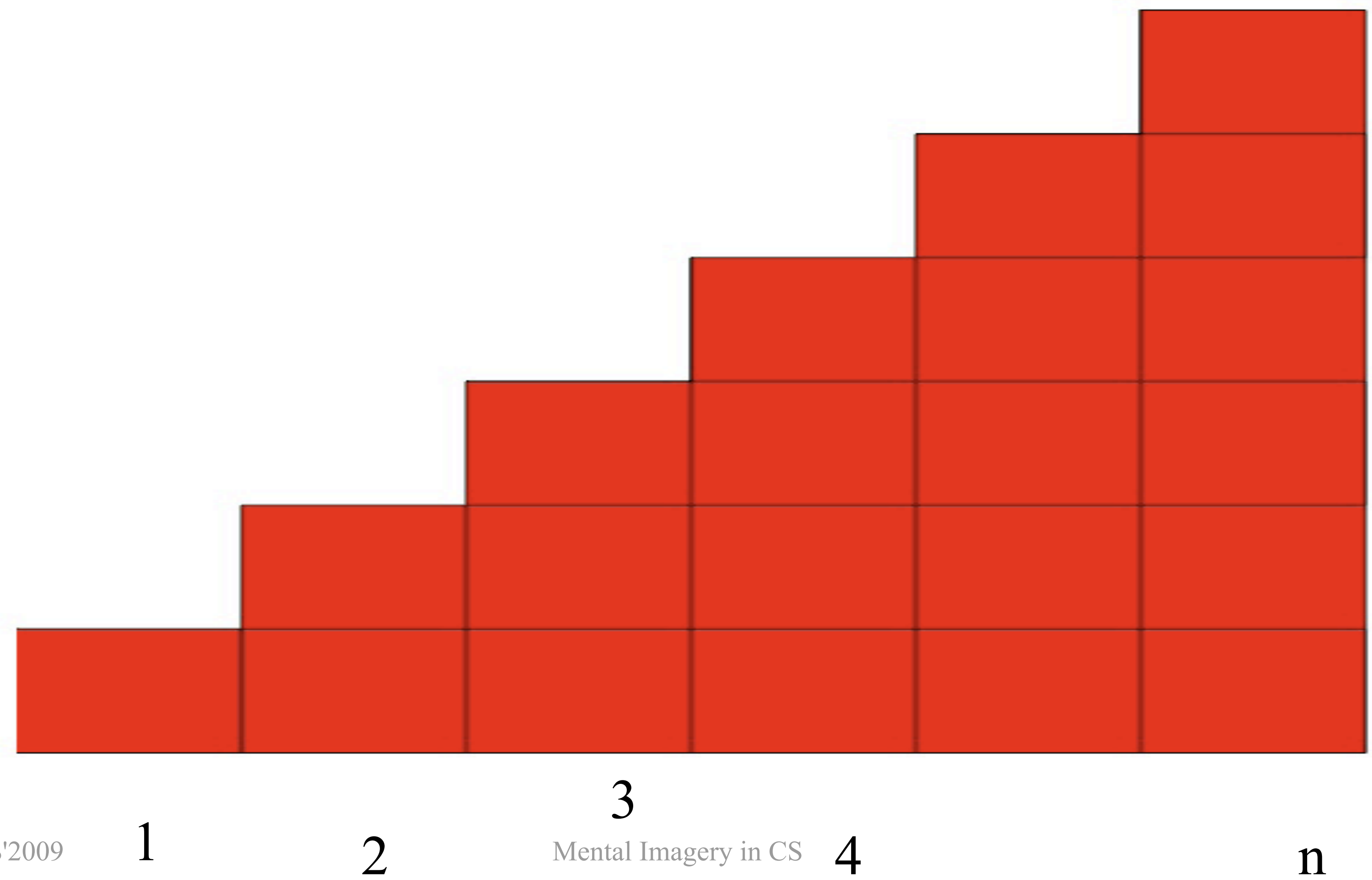
- Verbal/algebraic proof, playing with the formula (Gauss find)

  $S(n) = 1 + 2 + 3 + \ldots + (n - 1) + n$

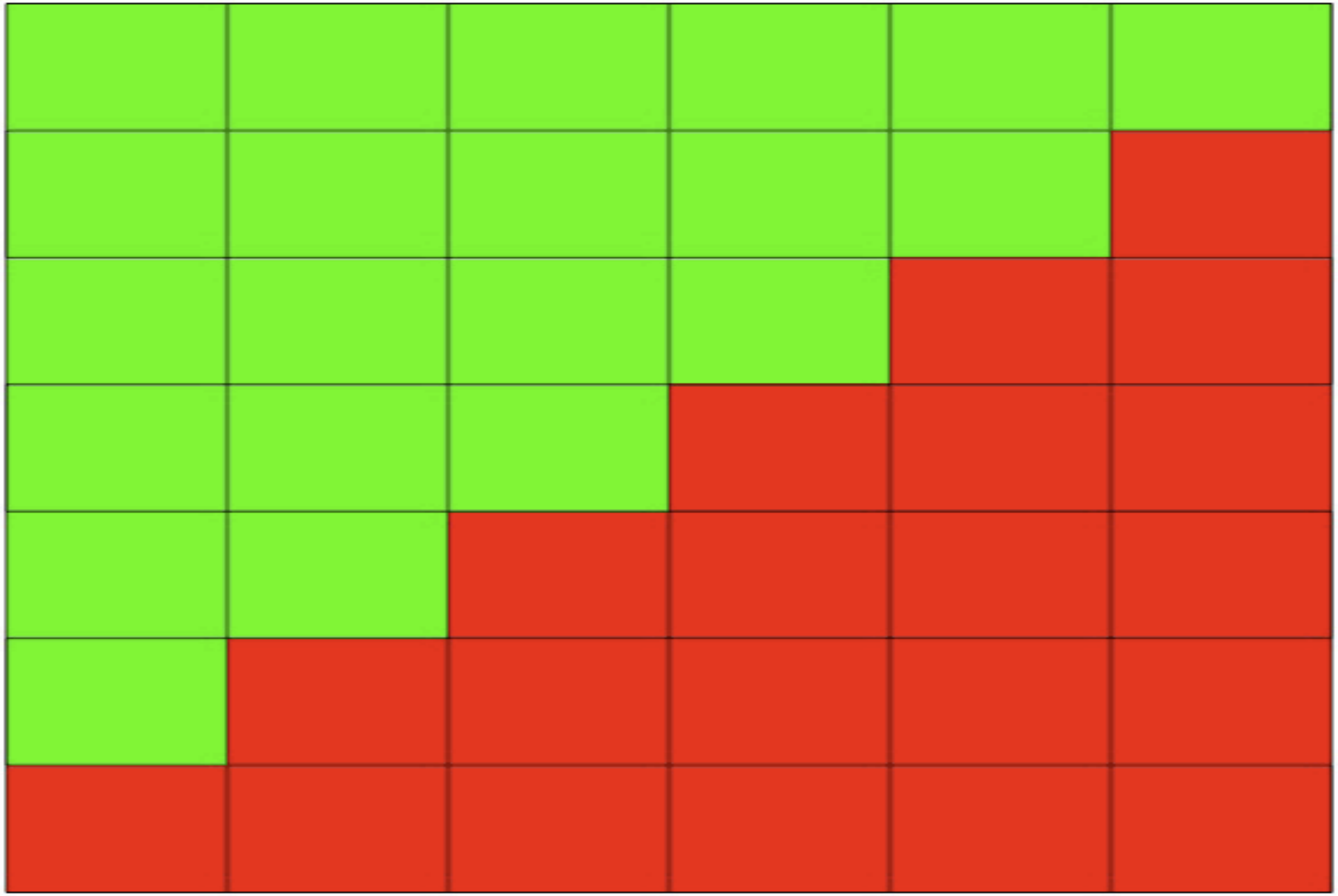
  $S(n) = n + (n - 1) + \ldots + 3 + 2 + 1$

  $2xS(n) = (n+1)+(n+1)+\ldots+(n+1) = n(n + 1) = n(n+1)$

Visual idea…

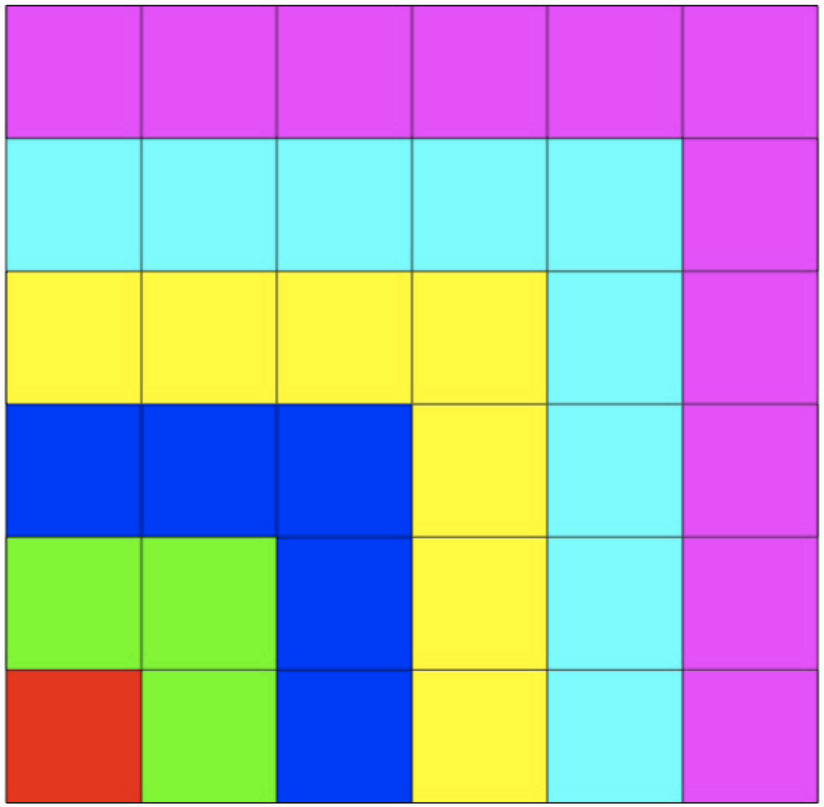


and proof



$$S(n) = 1 + 3 + 5 + \ldots + (2n - 1)$$

- Verbal: $S(n) = ?$ Easy induction for $S(n) = n^2$ but how to find $S(n) = n^2$?
- Image:



Types of definition: a function f is a …

- Verbal (≠types):
  - Triple f=(X,Y,x-->y) such that

    for all x,y,z (f(x)=y and f(x)=z) implies y=z
  - Subset f of XxY satisfying:
    1. for all x, there exists at most an y s.t. (x,y) ∈ f
    2. for all x,y,z, if (x,y) ∈ f and (x,z) ∈ f then y=z
  - Formula f(x,y) satisfying:

    for all x,y,z, (f(x,y) and f(x,z)) implies y=z

- Image:
  - Curb… but not every function is representable by a (visible) curb and some curbs are not functions
  - Graph: every node in X has at most a successor in Y (ok for finite graphs and if X=Y).

A function is…

- Mixte: verbal+image+movements
  - All the previous examples: words + images + feelings
  - Process which takes input x, works on x and produces output y
  - Algorithm computing f(x)
  - Other….

- Examples
  - Difficult to see: f(real\rational)=1 et f(rational)=0
  - Verbal, sequential view: g(n)=0 si T_n stops (in n steps) on input n.
  - Interesting to see: f(x)=x, f(x)=x^2, f'(x)=x,…

- Use mixted representations !

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Graphs and finite automata

Graph:

- relation (edges) between a set E of vertices.
- adjacency matrix
- picture of edges linking a finite number of vertices
- Abstract or concrete graph, labyrinth
- examples (metro,…)

Finite automaton:

- 5-uple (E,A,δ,e_0,F)
- (Abstract) labelled graph
- (Concrete) labelled graph: labyrinth with rooms, named one-way corridors, entrance and exit rooms
- A machine which produces outputs, which reads inputs, both.

Regular langages

- Recognized by an automaton: image-movements for seing words…

- Generated by a regular grammar : verbal-sequential, possible to add a tree (image)

- Described by a regular expression: verbal, static.

Pumping lemma

Lemma: *Let L be a regular language. There exists $k_L \in N$ such that any word $w \in L$, of length larger than k_L, can be factored $w = xuy$, with u non empty and $xu^ny \in L$ for all $n \in N$.*

Proof:

- from the regular expression ?
- From the regular grammar ?
- Better by using the Kleene Theorem for the bridge between algebra and machines and then by using the visual/kinesthesic representation of an automaton as a graph.

Bridges

- Kleene theorem: FA = REG, a bridge between
  - machines (automata) and algebra (languages)
  - concrete objects (graphs) and abstract objects (subsets)
  - images (graphs, machines) and words (formula)

- Büchi theorem: MSO = REG, a bridge between
  - Logics (verbal) and automata (graphs, machines)

Other bridges

- Ginsburg & Spanier: SL = Presburger logics
  - Geometry (visual), algebra (verbal), numbers (concrete objects) and logics (formula, equations, abstract words)
- Rabin, Comon, Wolper: Presburger Logics $\subseteq$ FA
  - Logics and automata (hence algorithmics for logics).
- REG(N^p)= Presburger logics
  - Algebra and logics and more or less complex bridges.

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Mental images from 2000 years

- Avant JC: Platon, Aristote, Epictète…
- 1600: Descartes
- 1700-1800: Locke, Hume, Berkeley, Kant,…
- 1890-1950: Husserl, Heidegger, Merleau-Ponty, Proust, Freud
- 1900-1930: Tichener, Binet (psycho. Introspective) : profils visuels, auditifs, moteurs
- 1910: Perky (conflit entre perception et évocation)
- 1930: Pavlov, Skinner, Watson (comportementalisme)
- 1933: Sémantique générale (Korzibsky)
- 1936: Turing, thèse de Church
- 1940-60: Ecole de Piaget
- 1956: Naissance des sciences cognitives, Miller (7+2), Galanter, Pribram
- 1956-60: Naissance des thérapies cognitives: Beck et Ellis (USA)
- 1970-1980: Paivio, Kosslyn, Pinker, Denis (double codage V/A)
- 1985: Gardner (intelligences multiples)
- 1986: Baddeley (Calepin visuo-spatial et boucle phonologique, attention)
- 1995: Damasio, Goleman
- 2009: Berthoz, Dehaene, Mellet,…

Aristote

« Jamais l’âme ne pense sans image »

« Never the mind thinks without image »

Descartes

Prefers words than image and created the analytic geometry, i.e. solving geometric problems by solving equations !

Einstein

No words during his thinking
And a lot of MI in his texts

« les mots et le langage, écrits ou parlés, ne semblent pas jouer le moindre rôle dans le mécanisme de ma pensée »

Poincaré

The converse !

Bourbaki

Words > images

Axiomatisation, …
No image in the books !

Benoît Mandelbrot

Images > words

He leaved the ENS because he felt the place too much verbal and he created the geometry of fractals

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What is a mental representation?

Repetition is necessary…

- An image, a movement, a sound, linked to a mathematical concept or proof
- Several types of representations, adapted to various people
  - Visual, visio spatial: mental images or designs
  - Auditive, verbal, phonologic: a sound, a poetry, a sentence,...
  - Kinesthetic: a movement, feeling, emotion,…

I understanding when I am able…

– To manipulate symbols and words (computation, rewriting rules, apply knowledge,…)

And

– To create and manipulate mental objects through MI.

And

– To make connections between both !

An researcher

- Is able to create MR and to manipulate them
- Finds pleasure with/in his mental world
- Mental world may become very important then he sometimes ignores external world (mutex)
- May ignore that other people are not like her.

What is the use of a mental representation?

- Very important to get an intuition, to understand, to remember
- Good representations are hard to find so share them !
- One representation is only useful for a part of students; one needs several representations of various types.
- Good representations often need some effort to be used
- Representations do NOT replace algorithms and methods; they help to learn and use them

« Observations »

- Understand ≠ memorise
- Understand a definition = to build a well-adapted representation, to manipulate it

(Create, test, complete our representations)

- well-adapted = multiples codings: verbal, visual, auditive, feeling, movement, kinesthesic, parameters of attention.
- Understand a proof ≠ follow the proof line after line + build a well-adapted MR
- Prove = create the proof
- Teach = give few MR

Training

- 2h: a seminar
- 2 days to train a particular subject: MI, pedagogical communication, emotions, memory, motivation,…
- 20 days (janv. 09 - sept. 09): ACTA

Thank you

Conference: 11th ICME'2008, International Congress on Mathematical Education, Mexico.

Journal: IJMEST'2009.

Texts in http://plato.stanford.edu/

- http://plato.stanford.edu/entries/mental-imagery/
- http://plato.stanford.edu/entries/mental-representation/

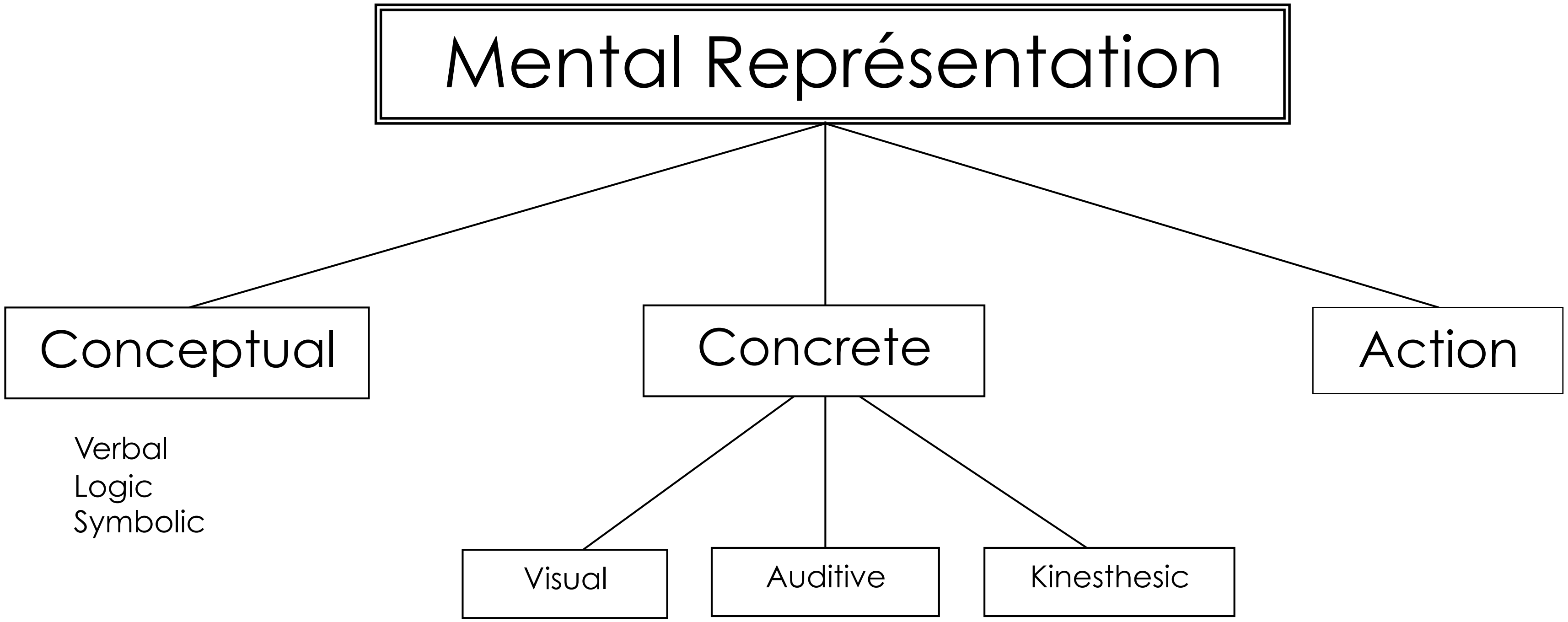
Jacques Hadamard, a pre-cognitive psychologist

« essai sur la psychologie de l'invention dans le domaine mathématique »

- 1945, Princeton.

Numberous connections between CS and psycho

- Virtual perception may replace/allow to develop MI for students without good MR.
- Turing created TM partially with introspection (multiplication ?) and cognitive psychology used CS (in 1960).
- **Extending the Vision: Images in History,** Colson, F. and Hall, W. (1990)
- Bases neurocognitives des stratégies de navigation chez l'Homme, PhD thesis under supervision of Berthoz
  - Allocentré: hypocampe droit, space
  - Egocentrée: hypo gauche, time sequence
  - Gps: good collaboration between words and images
- Virtual therapy….
- Brain-computer…


Mental Représentation
Conceptual
Concrete
Action
Verbal
Logic
Symbolic
Visual
Auditive
Kinesthesic